

centre of the 8 is placed over any mark on the horizontal line numbered, say, 9, the point on the periphery of the 8 numbered 9 must be superposed on the point on the sinuous curve also numbered 9.

The description of the sinuous line is a simple case of mechanical drawing, and presents no difficulty. By varying the harmonic composition of the figure of eight and the rate of undisturbed flow of the water, an infinite number of different individual curves can be produced which are all covered by the same generic specification. It is an interesting occupation, in leisure moments, to compose curves of this kind and to compare them with those traced by actual rivers on the face of the earth.

J. Y. BUCHANAN.

It is not difficult to show the character of the flow at the bottom of a small river. For several years I have taken my students along the course of the river Fender near Birkenhead, and we have conducted experiments which confirm the laws of bottom flow first pointed out by Thomson. At first we put down tubes containing coloured liquids, and the stream-line motion was very clearly shown by lines of colour. Later, I have employed lump sugar soaked in a strong alcoholic solution of magenta. On placing one of these cubes at the outer bend of a curve—the “turnpool”—it is found that the water there is almost stagnant. Gradually an aureole of coloured water forms round the sugar as it dissolves, and this slowly creeps across the stream towards the inner bend. The advantage of this method is that the coloured sugar is several minutes in dissolving, and it is very easily carried about.

For surface flow I have found mahogany sawdust to be the best, as it approaches water in density, and the fine particles are not influenced by air currents.

Although in measuring the surface flow the line of maximum velocity is usually more eccentric than the middle line of the stream, there are cases where the quickest flow is near the inner bend.

In a small experimental river in my laboratory I can produce both effects at will. A river is always tending towards a definite adjustment of its parts to correspond with the characteristics of its flow. The floor becomes graded by the filling of hollows and the removal of obstacles, and the swings become regular and rhythmic like the swings of a pendulum. This condition is seldom found except in the flood plains, and I presume this is the special case referred to by Sir Oliver Lodge.

In the ungraded part many exceptional and interfering circumstances come into play. I have noticed in experimenting with my laboratory river that when the stream has become perfectly adjusted the line of maximum flow is towards the outer curves, but if any disturbing cause is introduced, such as an increase or decrease in the quantity of water flowing in the channel, a variation in the slope of the bed giving a more rapid or gentler fall, or the introduction of an obstacle to form narrows, the normal characteristics of the flow are disturbed, and the maximum flow may be on the inner curve or more violently bent to the outer curve. This is determined by the changed conditions and the tendency of the stream to readjust itself. Prof. Thomson's model, not dealing with graded conditions, may easily have produced the abnormal effects he describes.

A comparison of the flow of a river with that of a glacier shows more points of similarity than most people suppose. In a curved glacier such as the Findelen, the surface at the outer bend is higher than at the inner bend, and the inner bend is always marked by a “toe-cap” moraine like the shallows on the inner curve of a river. It is only reasonable to suppose that this is the result of a cross current under the glacier, such as can be demonstrated in rivers. Moreover, we have in glaciers the phenomena of whirlpools and eddies where tributaries join the main stream.

The phenomena of flow are practically the same whether the medium is solid, liquid, or gaseous. The essential feature of flow is *shearing*. In a stream the surface layers shear over the lower, the mid-stream portion shears

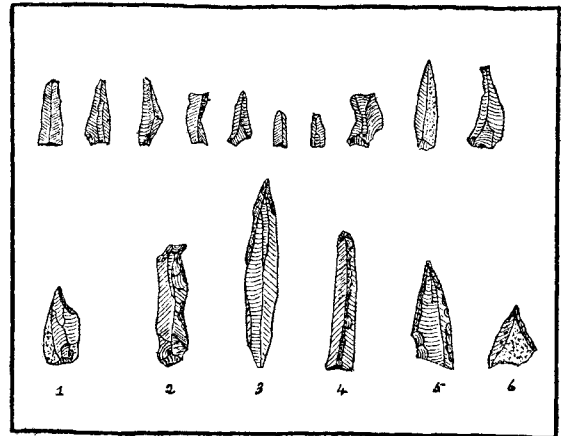
through the lateral parts, and in a meandering course momentum impels the water towards the outer bends and shears it round the slower moving water in the inner bend. So in glaciers and even in solid rocks flowing under earth stresses the same laws apply. The only difference lies in the unit of shear. In the case of liquids and gases this is extremely small, whereas in glaciers it is usually the “Kugel” which gives rise to the corn structure in glacial ice. In rocks the unit varies from masses of gigantic size to others of very small dimensions. This, perhaps, may be regarded as a very crude conception of the meaning of flow, but I have found it useful in giving students a graphic idea of the complicated movements in rivers and glaciers.

The University, Liverpool.

J. LOMAS.

Small Flint Implements from Bungay.

THE small flint implements figured in the accompanying drawing were found in a sandy hollow about 2 feet deep at Bungay, in Suffolk. The sand in this hole was littered with minute flakes; in a few minutes I picked up between fifty and sixty, of which the figured ones are typical examples. I hesitate to describe the implements as “pigmy flints,” because their fine secondary chipping is not confined to the thicker edge or “back” of the flakes, but, judging from photographs I have seen, they closely resemble some pigmies found recently near Brighton by Mr. H. S. Toms. So far as the untrimmed flakes are concerned, it is impossible to distinguish them from typical



Small flint flakes and implements from Bungay. Two-thirds actual size.

pigmy flakes, while the trimming of implements 3 and 5 is identical with that of the pigmies.

In consequence of nearly all the English pigmies having been found on the surface of the ground, it has been impossible to say with any confidence whether they belong to the Neolithic, Bronze, or Early Iron period. In view of this, it is interesting to know that the small flakes and implements from Bungay were found in association with a polished axe of grey flint, a black flint lance-head of very delicate workmanship, one of the rare and finely chipped triangular “knives,” and some small convex scrapers showing very delicate secondary chipping. These implements were found in the same sandy hole when the small implements were discovered, and from an examination of the sides of the hollow it was evident that they all came from what might be called a “Neolithic floor” about 18 inches from the surface of the ground. Nowhere on the surface of the surrounding ground could I find a single flake or implement, and if the ground had not been disturbed in order that a small quantity of sand might be carted away, not one of the implements would have been brought to light. As it happened, they were all found within an area of about six square yards. Some

small bones found on the same site have been identified as those of a girl or a small woman.

The makers of the small flint implements evidently had their home or their "workshop" on a sandy knoll only a few feet above the level of the marshes of the Waveney Valley. On this knoll and a neighbouring one there are some saucer-shaped depressions in the ground very suggestive of hut-circles.

Lowestoft.

W. A. DUTT.

Graphical Interpolation.

SIR GEORGE DARWIN has directed attention (*Mess. of Math.*, 1877; *Phil. Trans.*, A, 1891; "Collected Works," vol. i., p. 319) to the problem of interpolating values of

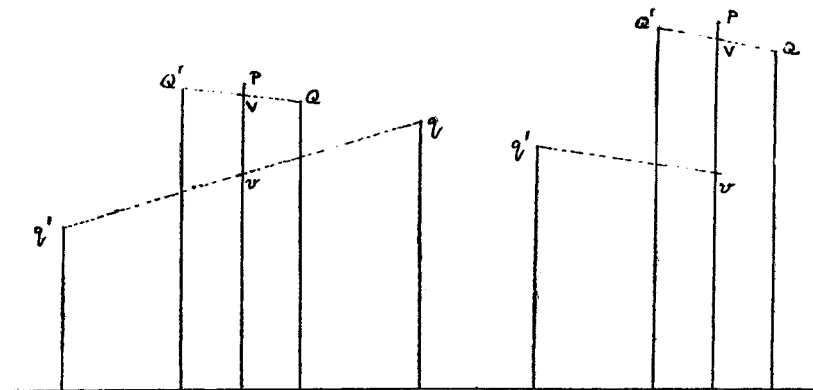


FIG. 1.

FIG. 2.

a function at points each half-way between two consecutive points of an equidistant set, e.g. for determining probable half-hourly values when the hourly ones are found from observations. Let q' , Q' , Q , q be four points (Fig. 1) with equidistant ordinates u' , U' , U , u . It is required to find P where the graph through these four points cuts the ordinate half-way between Q and Q' . By taking the origin on the half-way ordinate and writing the function as

$$y = a + bx + cx^2 + dx^3 + \dots,$$

we find that if we neglect terms beyond x^3 , then

$$a = \frac{U' + U}{2} + \frac{1}{8} \left(\frac{U' + U}{2} - \frac{u' + u}{2} \right).$$

A rule for determining the point P is accordingly:—join QQ' , qq' and let them cut the central ordinate in V , v respectively, then P lies in vV produced, and $PV = \frac{1}{3}Vv$. This rule, although theoretically identical, is simpler in form than that discovered by Sir George Darwin, and seems to be safer, especially near a point of inflexion. It may be worth noticing that in the special case where QQ' and qq' are parallel, the cubic reduces to a parabola, and the rule for finding P is involved in the relation $PV : Pv = QV^2 : qv^2 = 1 : 9$. At the beginning and end of the series the rule breaks down, but it can be adapted by assuming the parabolic form for the first and last arcs. In the latter case q is indeterminate, and $q'v$ must be drawn parallel to $Q'Q$ (Fig. 2).

In the diagram (Fig. 3) the rule is applied to an example in which the assumption that the function can be expressed

as a power series is scarcely justifiable, but it will be seen that it makes it easy to draw a smooth curve through the points Q .

F. J. W. WHIFFLE.

Merchant Taylors' School, E.C.

Reflection of Polarised Light.

SOME recent correspondence (vol. lxxvi., p. 637) having directed attention to an error in Preston's "Theory of Light," I venture to send notice of another error in the same work (see article 158). The same error will be found in Prof. Tait's article on light (see p. 611, vol. xiv., of the "Encyclopædia Britannica"), and is repeated in his text-book on light (article 271).

I sent word of the error to the late Sir George Stokes, who expressed himself astonished at it, and said he would look into the matter; but I did not hear from him again, as his letter to me (dated September 19, 1902) was written only five months before his death.

Let the planes of two thin plates of ordinary glass, A and B , be parallel, so that light, which has been completely plane-polarised by reflection from A , falls at the polarising angle upon B . Preston states that this light will be wholly reflected from B , whilst Tait states that this light will be reflected, almost without loss, from B .

As a matter of fact, if we represent by unity the intensity of the polarised beam incident upon B , then the intensity of the light reflected from B will be represented by about $\frac{2}{3}$, and this takes into account both surfaces of B . The remainder of the beam, about $\frac{1}{3}$, is transmitted. To reflect the whole of the incident beam an infinite number of plates would be

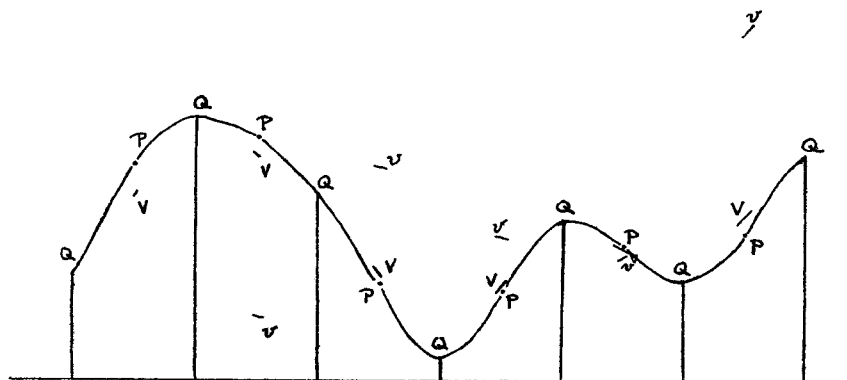


FIG. 3.

required, and the glass would have to be perfectly transparent.

Both authors state correctly that, when the plane of reflection of B is perpendicular to that of A , and the polarised light from A falls at the polarising angle on B , then practically none of this light will be reflected from B .

I therefore think that the mistake arose from accidentally supposing that the total want of reflection in the second case should be balanced, as it were, by a complete reflection in the first case.

C. T. WHITMELL.

Invermay, Hyde Park, Leeds.